Problem 14, §6.4

Prove that for small x

$$\frac{1}{x}\log(1+x) = 1 - \frac{x}{2} + \frac{x^2}{3} + \dots,$$

and hence that

$$(1+x)^{\frac{1}{x}} = e(1 - \frac{x}{2} + \frac{7}{12}x^2 + \dots).$$

Hence prove that

$$\frac{e - (1+\frac{1}{n})^n}{1/n} \to \frac{e}{2} \text{ as } n \to \infty.$$